ETH Zürich, D-INFK
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Diskrete Mathematik

Exercise 5

5.1 Family relations ($\star\star$)

Consider the set of all people (both living and already dead) and different family relations on this set: *id* (identity), *if* (is the father of), *im* (is the mother of), *ip* (is a parent of) and *ic* (is a child of).

a) Express each of the following relations, using the relations above.

- **i)** x *iggf* y $:\Longleftrightarrow$ x is a great grandfather of y
- **ii)** x *ihs* y $:\Longleftrightarrow$ x is a half sibling of y (i.e., x and y have exactly one common parent)
- **iii)** x *ico* y $:\Longleftrightarrow$ x is a cousin of y (i.e., x and y are not siblings but have a common grandparent)

b) What is the relationship between the relations *ic* $\circ$ *ic* $\circ$ *ip* $\circ$ *ip* and *ic* $\circ$ *ip* $\circ$ *ic* $\circ$ *ip*? Are they the same relation or is one of them a subset of the other?

5.2 Operations on relations ($\star\star$)

Let us consider the relations $<$, $|$ and $\equiv_2$ on the set of natural numbers $\mathbb{N} \cup \{0\}$. For each of the following relations on $\mathbb{N} \cup \{0\}$, decide whether it is reflexive, symmetric or transitive. Justify your answers.

a) $< \circ \mid$

b) $\mid \cup \equiv_2$

c) $\mid \cup \mid^{-1}$

5.3 Properties of relations *(7 Points)*

a) ($\star\star$) For the relation $\rho = \{(1,4),(2,1),(2,3),(4,2)\}$ on the set $\{1,2,3,4\}$, determine the relations ρ^3 and ρ^*. Describe ρ^3 using the set representation, while ρ^* using matrix representation. *(2 Points)*

b) ($\star\star$) Prove or disprove the following statement: for any set A, if a relation σ on A is not reflexive, then the relation σ^2 is also not reflexive. *(2 Points)*

c) ($\star\star\star$) Prove or disprove the following statement: for any set A, if relations σ and ρ on A are antisymmetric, then so is the relation $\sigma \cap \rho$. *(3 Points)*

5.4 A false proof ($\star\star$)

Consider a non-empty set A and a symmetric and transitive relation $\rho \neq \emptyset$ on A.

a) The following proof shows that ρ is always reflexive. Find the mistake in this proof.

Proof: We show that ρ is reflexive, that is that for any x, we have $x \; \rho \; x$. Let $x \in A$. Further, let $y \in A$ be such that $x \; \rho \; y$. Since ρ is symmetric, it follows that $y \; \rho \; x$. Now we have $x \; \rho \; y$ and $y \; \rho \; x$. Hence, by the transitivity of ρ, it follows that $x \; \rho \; x$.

b) Show that the above statement is indeed false, that is, prove that ρ is not always reflexive.

5.5 Equivalence Relations

For an $x \in \mathbb{R}$, let us define the following relation $\sim$ on $\mathbb{R}^2$:

$$(a,b) \sim (c,d) \quad :\Longleftrightarrow \quad \big((a-x)^2+b^2\big)\big((a+x)^2+b^2\big) - \big((c-x)^2+d^2\big)\big((c+x)^2+d^2\big) = 0$$

a) ($\star\star$) Show that $\sim$ is an equivalence relation.

b) ($\star\star$) Describe geometrically the equivalence classes $[(a,b)]$.

Due on 23. October 2017.
Exercise 5.3. will be corrected.