

Diskrete Mathematik

Exercise 6

6.1 Partial order relations

a) (★) Consider the poset $(\mathbb{N} \setminus \{0\}, |)$. Which of the following pairs are comparable?

- i) 11, 12 ii) 4, 6 iii) 5, 15 iv) 42, 42

b) (★) Consider the set $(\mathbb{N} \setminus \{0\}) \times (\mathbb{N} \setminus \{0\})$ with lexicographic order defined by the divisibility relation $|$. Determine all elements smaller than $(2, 5)$.

c) (★) Prove or disprove the following statement: $(\{1, 3, 6, 9, 12\}, |)$ is a lattice.

d) (★★) Let (A, \preceq) be any poset. Prove or disprove the following statement: (A, \preceq^{-1}) is a poset.

6.2 Hasse diagram (★)

Consider two posets $(\{1, 2, 3\}; \leq)$ and $(\{1, 2, 3, 5, 6, 9\}; |)$. Draw the Hasse diagram of each of these posets. Determine all least, greatest, minimal and maximal elements.

6.3 Lexicographic order (★★★)

(8 Points)

Prove Theorem 3.11 from the lecture notes.

6.4 Countability

a) Determine whether the following sets are countable or uncountable. Prove your answers.

i) (★) The set of all Java programs.

ii) (★★) The set A of all infinite sequences over $\{0, 1, \dots, 9\}$.

iii) (★★) The set C of all points on the unit circle, that is $C := \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$.

iv) (★★★) The set of equivalence relations on \mathbb{N} .

b) (★★★) You have to sink an submarine. The submarine moves with the constant speed $v \in \mathbb{Z}$. At a given point in time $t \in \mathbb{N}$, it is located at position $v \cdot t + s_0$, where $s_0 \in \mathbb{Z}$ is its starting point. You do not know the values v and s_0 . At each point in time you can fire a torpedo to one position $s \in \mathbb{Z}$. If at this moment the submarine can be found exactly at position s , it sinks. Is there a strategy that allows to sink the submarine in finite time?

6.5 A property of sets (*)**

Show that there exists an infinite number of sets X such that $X \subseteq \mathcal{P}(X)$.

Hint: You can use induction.

Due on 30. October 2017.
Exercise 6.3. will be corrected.