ETH Zürich, D-INFK
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Prof. Ueli Maurer
Marta Mularczyk

Diskrete Mathematik

Exercise 2

This exercise is not graded.

2.1 A Proof ($\star\,\star$)

Claim: 1 is the largest natural number.

Proof: Let n be the largest natural number.
$\Rightarrow n^2 \leq n.$
$\Rightarrow n(n-1) = n^2 - n \leq 0$
$\Rightarrow 0 \leq n \leq 1$
$\Rightarrow n = 1.$

Find the mistake.

2.2 Propositional Logic

Let A be the proposition "The monkey sits on the palm tree." and let B be the proposition "The monkey has a banana.".

a) ($\star$) How would you interpret the following formulas?

i) $F_1 := B \wedge \neg A$ **ii)** $F_2 := (A \wedge B) \vee (\neg A \wedge \neg B)$

b) ($\star$) Using only the propositions A, B and logical operators, write down formulas corresponding to the following sentences:

i) F_3: "The monkey neither sits on the palm tree nor has a banana."

ii) F_4: "The monkey either has a banana or sits on the palm tree (but not both)."

c) ($\star\,\star$) For both formulas F_3, F_4, write down their negations (both as sentences and formally as formulas).

2.3 Function Tables and Equivalence

a) ($\star$) Compute the function table for the following formula:

$$(B \to C) \to \big(\neg(A \to C) \wedge \neg(A \vee B)\big).$$

b) ($\star\,\star$) Give another formula that is equivalent to the formula from the subtask a). Try to make it as simple as possible.

c) ($\star\star\star$) Prove formally that the formula from your solution to the subtask b) is equivalent to the formula in a). Your proof should be in the form of a sequence of steps, where each step consists of applying the definition of $\rightarrow$, or one of the rules given in Lemma 2.1 of the lecture notes. You can also use the fact that for any formula F, we have $F \wedge \neg F \equiv \bot$, $F \wedge \bot \equiv \bot$ and $F \vee \bot \equiv F$.

2.4 Logical Consequence *(7 Points)*

Prove or disprove the following statements about formulas.

a) ($\star$) $A \wedge (A \rightarrow B) \models B$ *(2 Points)*

b) ($\star$) $A \rightarrow B \models \neg A \rightarrow \neg B$ *(2 Points)*

c) ($\star\star$) $(A \rightarrow B) \wedge (B \rightarrow C) \models (A \rightarrow C)$ *(3 Points)*

2.5 Satisfiability and Tautologies ($\star$)

For each of the formulas below, determine whether it is satisfiable or unsatisfiable and whether it is a tautology or not. Justify your answers.

a) $(A \vee B) \wedge \neg A$

b) $\big((A \rightarrow B) \wedge (B \rightarrow C)\big) \wedge \neg(A \rightarrow C)$

2.6 Knights and Knaves ($\star\star\star$)

You find yourself alone on a strange island with only two types of inhabitants: knights and knaves. The knights always tell the truth, while the knaves always lie. From the outside, both groups look exactly the same and you cannot distinguish one from the other.

You have lost your way and you come to a fork in the road. You know that one of the roads leads to a deadly jungle, while the other will take you to a friendly village. You see an islander standing at the fork. He is willing to answer only one question and his answer can only be "Yes" or "No". What question would you ask?

Try to use propositional logic, in order to solve this exercise.

Due on 1. or 2. October 2018.
Exercise 2.4 will be corrected.